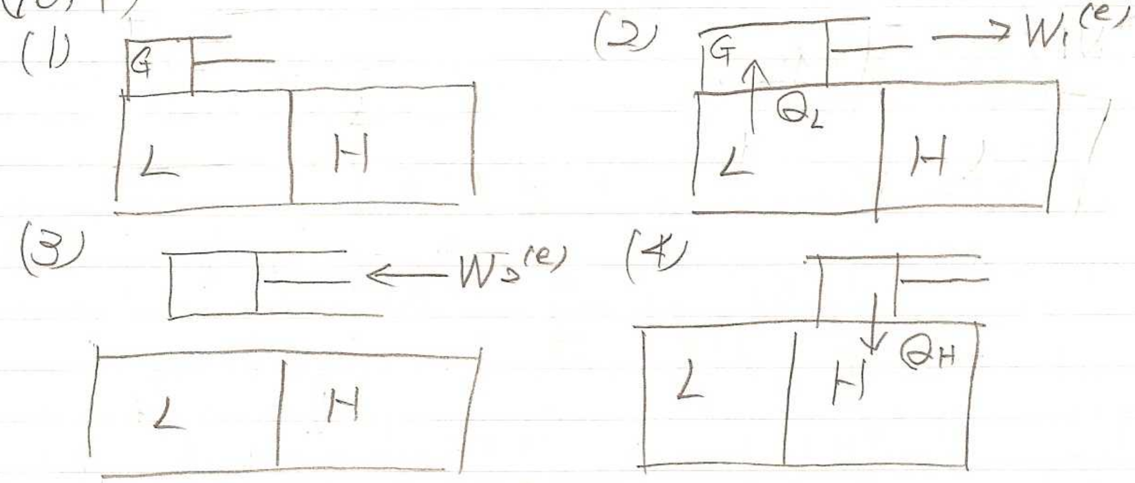
2. $\eta_{冷} \leq \frac{27}{32-27} = 5.4$

3. $\eta_{冷} \leq \frac{22}{32-22} = 2.2$

(10.4)



LとGを図10.6のLと見なす。

$Q = Q_H$

$W^{(e)} = W_1^{(e)} + W_2^{(e)}$

(10.5) 必要なし

(11.1)

1. $f(x) = \begin{cases} x & (0 < x < 1) \\ 1 & (1 < x < 2) \\ x & (2 < x) \end{cases}$

$f'(x) = \begin{cases} 1 & (0 < x < 1) \\ 0 & (1 < x < 2) \\ 1 & (2 < x) \end{cases}$

$f'(x) \geq 0$ より下に凸

No. Date

2. $p = f'(x) = \begin{cases} x & (0 \leq x < 1,\ 2 < x) \\ 1 & (1 < x < 2) \end{cases}$

$x=0$ のとき $p=0$ $g(p) = xp - f(x) = 0$

$0 < x < 1$ で

$g(p) = xp - f(x) = p^2 - \frac{p^2}{2} = \frac{1}{2}p^2 \quad (0<p<1)$

$1 < x < 2$ で

$g(p) = xp - f(x) = x - (x - \frac{1}{2}) = \frac{1}{2} \quad (p=1)$

$2 < x$ で

$g(p) = xp - f(x) = \frac{1}{2}p^2 + \frac{1}{2} \quad (p>2)$

$x=1$ で $f'(x-0) = 1,\ f'(x+0) = 1 \ \therefore p=1$

$g(p) = xp - f(x) = \frac{1}{2}$

$x=2$ で $f'(x-0) = 1,\ f'(x+0) = 2 \ \therefore 1 \leq p \leq 2$

$g(p) = xp - f(x) = 2p - \frac{3}{2}$

$\therefore g(p) = \begin{cases} \frac{1}{2}p^2 & (0 \leq p \leq 1) \\ 2p - \frac{3}{2} & (1 \leq p \leq 2) \\ \frac{1}{2}p^2 + \frac{1}{2} & (p > 2) \end{cases}$

(11.2)

$g'(p) = \begin{cases} p & (0 \leq p \leq 1,\ p > 2) \\ 2 & (1 \leq p \leq 2) \end{cases}$

$g''(p) = \begin{cases} 1 & (0 \leq p \leq 1,\ p > 2) \\ 0 & (1 \leq p \leq 2) \end{cases}$

$\therefore g''(p) \geq 0$ $\therefore$ 下に凸

(11.3)

$$x = g'(p) = \begin{cases} p & (0 \leq p \leq 1) \\ 2 & (1 \leq p \leq 2) \\ p & (p > 2) \end{cases}$$

$p=0$ のとき $x=0$, $f(x) = px - g(p) = 0$

$0<p<1$ のとき

$f(x) = px - g(p) = \frac{1}{2}x^2 \quad (0<x<1)$

$1<p<2$ のとき

$f(x) = p \cdot 2 - g(p) = \frac{3}{2} \quad (x=2)$

$p>2$ のとき

$f(x) = px - g(p) = \frac{1}{2}x^2 - \frac{1}{2} \quad (x>2)$

$p=1$ のとき $g'(p-0) = 1$, $g'(p+0) = 2$

$f(x) = 1 \cdot x - g(1) = x - \frac{1}{2} \quad (1 \leq x \leq 2)$

$p=2$ のとき $g'(p-0) = 2$, $g'(p+0) = 2$ $\therefore x = 2$

$f(x) = 2 \cdot 2 - g(2) = \frac{3}{2}$

$$\therefore f(x) = \begin{cases} \frac{1}{2}x^2 & (0 \leq x < 1) \\ x - \frac{1}{2} & (1 \leq x \leq 2) \\ \frac{1}{2}x^2 - \frac{1}{2} & (x > 2) \end{cases}$$

(11.4)

1. $f'(x) = \begin{cases} x & (x<1) \\ 1 & (1 \leq x < 2) \\ x & (2 \leq x) \end{cases}$ $\quad f''(x) = \begin{cases} 1 & (x<1) \\ 0 & (1 \leq x < 2) \\ 1 & (2 \leq x) \end{cases}$

$f''(x) \geq 0$ より下に凸

No. Date

2. $p = f'(x) = \begin{cases} x & (x<1) \\ 1 & (1 \leq x < 2) \\ x & (2 \leq x) \end{cases}$

(i) $x < 1$

$g(p) = xp - f(x) = \frac{1}{2}p^2 - 1 \quad (p<1)$

(ii) $1 \leq x < 2$

$g(p) = xp - f(x) = -\frac{1}{2} \quad (p=1)$

(iii) $x = 2 \quad f'(x-0) = 1,\ f'(x+0) = 2$

$g(p) = xp - f(x) = 2p - \frac{5}{2} \quad (1 \leq p \leq 2)$

(iv) $x \geq 2$

$g(p) = xp - f(x) = \frac{1}{2}p^2 - \frac{1}{2} \quad (p \geq 2)$

$\therefore g(p) = \begin{cases} \frac{1}{2}p^2 - 1 & (p \leq 1) \\ 2p - \frac{5}{2} & (1 \leq p \leq 2) \\ \frac{1}{2}p^2 - \frac{1}{2} & (p \geq 2) \end{cases}$

3. $g'(p) = \begin{cases} p & (p \leq 1) \\ 2 & (1 \leq p \leq 2) \\ p & (p \geq 2) \end{cases} \qquad g''(p) = \begin{cases} 1 & (p \leq 1) \\ 0 & (1 \leq p \leq 2) \\ 1 & (p \geq 2) \end{cases}$

∴下に凸

4. $x = g'(p) = \begin{cases} p & (p \leq 1) \\ 2 & (1 \leq p \leq 2) \\ p & (p \geq 2) \end{cases}$

Date . . No.

(i) $p<1$
$f(x) = px - g(p) = \frac{1}{2}x^2+1 \quad (x<1)$

(ii) $p=1 \quad g'(p-0)=1,\ g'(p+0)=2$
$f(x) = px - g(p) = x + \frac{1}{2} \quad (1 \leq x \leq 2)$

(iii) $1<p\leq 2$
$f(x) = p\cdot 2 - g(p) = \frac{5}{2} \quad (x=2)$

(iv) $p\geq 2$
$f(x) = px - g(p) = \frac{1}{2}x^2+\frac{1}{2} \quad (x\geq 2)$

$$\therefore f(x) = \begin{cases} \frac{1}{2}x^2+1 & (x<1) \\ x+\frac{1}{2} & (1\leq x\leq 2) \\ \frac{1}{2}x^2+\frac{1}{2} & (x\geq 2) \end{cases}$$

(11.5) $g(p) = px - f(x)$, $\bar{g}(x) = px - \bar{f}(x)$ とおくと
$\bar{g}(x) = px - f(x) + f_0 = g(p) + f_0$
$g'(p-0) \leq x \leq g'(p+0)$ のとき

$$\bar{f}(x) = px - \bar{g}(x) \Leftrightarrow f(x) - f_0 = px - g(p) - f_0$$
$$\Leftrightarrow f(x) = px - g(p)$$

(12.1)
$-S = \frac{\partial F}{\partial T}$

$$= N\left(cR - \frac{S_0}{N_0} - R\ln\left[\left(\frac{T}{T_0}\right)^c\left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right]\right) - cRN$$

$$S = \frac{N}{N_0}S_0 + RN\ln\left[\left(\frac{T}{T_0}\right)^c\left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right]$$

$$\therefore T = T_0\left(\frac{V_0}{V}\right)^{\frac{1}{c}}\left(\frac{N}{N_0}\right)^{\frac{1}{c}}\exp\left[\frac{1}{cR}\left(\frac{S}{N} - \frac{S_0}{N_0}\right)\right]$$

No. Date

$$U = F + ST = cRNT$$

$$U_0 = cRNT_0 \quad \therefore T_0 = \frac{U_0}{cRN}$$

$$\therefore U = U_0 \frac{T}{T_0} = U_0 \left(\frac{V_0}{V}\right)^{\frac{1}{c}} \left(\frac{N}{N_0}\right)^{\frac{1}{c}} exp\left[\frac{1}{cR}\left(\frac{S}{N} - \frac{S_0}{N_0}\right)\right]$$

(12.2)

$$S = -\frac{\partial F}{\partial T} = \frac{N}{N_0} S_0 + RN \ln\left[\left(\frac{T}{T_0}\right)^c \left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right]$$

$$P = -\frac{\partial F}{\partial V} = \frac{NRT}{V}$$

$$\mu = \frac{\partial F}{\partial N}$$

$$= T\left(cR - \frac{S_0}{N_0} - R \ln\left[\left(\frac{T}{T_0}\right)^c \left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right]\right) + RT$$

$$= T\left((c+1)R - \frac{S_0}{N_0} - R \ln\left[\left(\frac{T}{T_0}\right)^c \left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right]\right)$$

(12.3)

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{k_B}{\varepsilon} \ln\left(1 + \frac{\varepsilon V}{\gamma U}\right)$$

$$\therefore U = \frac{\varepsilon V}{\gamma\left(exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)}$$

代入して

$$S(T, V, N) = \frac{k_B V}{\gamma}\left[\frac{\varepsilon}{k_B T\left(1 - exp\left(-\frac{\varepsilon}{k_B T}\right)\right)} - \ln\left(exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)\right]$$

$$F = U - TS = -\frac{\varepsilon V}{\gamma} + \frac{k_B T V}{\gamma} \ln\left(exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)$$

$$P = -\frac{\partial F}{\partial V} = \frac{\varepsilon}{\gamma} - \frac{k_B T}{\gamma} \ln\left(exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)$$

(12.4)

$$S = K(UVN)^{\frac{1}{3}}$$

$$U = \frac{S^3}{K^3 VN} \quad T = \frac{\partial U}{\partial S} = \frac{3S^2}{K^3VN} \quad \therefore S = \frac{1}{\sqrt{3}} K^{\frac{3}{2}} V^{\frac{1}{2}} N^{\frac{1}{2}} T^{\frac{1}{2}}$$

$$F = U - ST = -\frac{2S^3}{K^3VN} = -\frac{2}{3\sqrt{3}} K^{\frac{3}{2}} V^{\frac{1}{2}} N^{\frac{1}{2}} T^{\frac{3}{2}}$$

$$P = -\frac{\partial F}{\partial V} = \frac{1}{3\sqrt{3}} K^{\frac{3}{2}} V^{-\frac{1}{2}} N^{\frac{1}{2}} T^{\frac{3}{2}}$$

$$\mu = \frac{\partial F}{\partial N} = -\frac{1}{3\sqrt{3}} K^{\frac{3}{2}} V^{\frac{1}{2}} N^{-\frac{1}{2}} T^{\frac{3}{2}}$$

(12.5)

$$F = \frac{NT}{N_0T_0} F_0 - RNT \ln\left[\left(\frac{T}{T_0}\right)^c \left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right]$$ ((12.20)式より)

$$P = -\frac{\partial F}{\partial V} = \frac{RNT}{V}$$

$$G = F + PV = \frac{NT}{N_0T_0} F_0 - RNT \ln\left[\left(\frac{T}{T_0}\right)^c \left(\frac{V}{V_0}\right)\left(\frac{N_0}{N}\right)\right] + RNT$$

(12.6)

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{k_B}{\varepsilon} \ln\left(1 + \frac{\varepsilon V}{\gamma U}\right) \quad \therefore U = \frac{\varepsilon V}{\gamma\left(\exp\left(\frac{\varepsilon}{k_BT}\right) - 1\right)}$$

代入して

$$S = \frac{k_B V}{\gamma}\left(\frac{\varepsilon \exp\left(\frac{\varepsilon}{k_BT}\right)}{k_BT\left(\exp\left(\frac{\varepsilon}{k_BT}\right) - 1\right)} - \ln\left(\exp\left(\frac{\varepsilon}{k_BT}\right) - 1\right)\right)$$

$$F = U - ST = -\frac{\varepsilon V}{\gamma} + \frac{k_BTV}{\gamma} \ln\left(\exp\left(\frac{\varepsilon}{k_BT}\right) - 1\right)$$

$$\frac{\partial F}{\partial V} = -\frac{\varepsilon}{\gamma} + \frac{k_BT}{\gamma} \ln\left(\exp\left(\frac{\varepsilon}{k_BT}\right) - 1\right) = \frac{F}{V}$$

$$G = F - \frac{\partial F}{\partial V} V = 0$$

(12.7)

$$\frac{\partial^2 F}{\partial V \partial T} = \frac{\partial^2 F}{\partial T \partial V} \Leftrightarrow \frac{\partial}{\partial V}(-S) = \frac{\partial}{\partial T}(-P) \Leftrightarrow \underline{\frac{\partial S}{\partial V} = \frac{\partial P}{\partial T}}$$

$$\frac{\partial^2 G}{\partial P \partial T} = \frac{\partial^2 G}{\partial T \partial P} \Leftrightarrow \frac{\partial}{\partial P}(-S) = \frac{\partial V}{\partial T} \Leftrightarrow \underline{-\frac{\partial S}{\partial P} = \frac{\partial V}{\partial T}}$$

$$\frac{\partial^2 H}{\partial P \partial S} = \frac{\partial^2 H}{\partial S \partial P} \Leftrightarrow \underline{\frac{\partial T}{\partial P} = \frac{\partial V}{\partial S}}$$

(13.1)

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{k_B}{\varepsilon} \ln\left(1 + \frac{\varepsilon V}{\gamma U}\right)$$

$$\therefore U = \frac{\varepsilon V}{\gamma\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)}$$

$$C_V = \frac{\partial V}{\partial T} = \underline{\frac{\varepsilon^2 V \exp\left(\frac{\varepsilon}{k_B T}\right)}{\gamma k_B T^2 \left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)}} \;//$$

$$c_v = \frac{C_V}{N} = \underline{\frac{\varepsilon^2 V \exp\left(\frac{\varepsilon}{k_B T}\right)}{\gamma N k_B T^2 \left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)}} \;//$$

(13.2)

$$F^{(1)} = -\frac{\varepsilon V^{(1)}}{\gamma} + \frac{k_B T V^{(1)}}{\gamma} \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)$$

$$F^{(2)} = -\frac{2}{3\sqrt{3}} K^{\frac{3}{2}} (V^{(2)})^{\frac{1}{2}} (N^{(2)})^{\frac{1}{2}} T^{\frac{3}{2}}$$

$$\begin{aligned}\tilde{F} &= F^{(1)} + F^{(2)} \\ &= -\frac{\varepsilon(V - V^{(2)})}{\gamma} + \frac{k_B T (V - V^{(2)})}{\gamma} \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right) \\ &\quad - \frac{2}{3\sqrt{3}} K^{\frac{3}{2}} (V^{(2)})^{\frac{1}{2}} (N^{(2)})^{\frac{1}{2}} T^{\frac{3}{2}}\end{aligned}$$

$$\frac{\partial \tilde{F}}{\partial V^{(2)}} = \frac{\varepsilon}{\gamma} - \frac{k_B T}{\gamma} \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right) - \frac{1}{3\sqrt{3}} K^{\frac{3}{2}} (V^{(2)})^{-\frac{1}{2}} (N^{(2)})^{\frac{1}{2}} T^{\frac{3}{2}}$$

$\frac{\partial \tilde{F}}{\partial V^{(2)}} = 0$ のとき

$$\underline{V^{(2)} = \frac{1}{27} \gamma^2 K^3 T^3 N \left(\varepsilon - k_B T \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)\right)^{-2}} \;//$$

$$V^{(1)} = V - V^{(2)}$$

$$= V - \frac{1}{27}\gamma^2 K^3 T^3 N\left(\varepsilon - k_B T \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)\right)^{-2}$$

(13.3) $P^{(1)} = P^{(2)}$ より

$$-\frac{\varepsilon}{\gamma} + \frac{k_B T}{\gamma}\ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right) = -\frac{1}{3\sqrt{3}} K^{\frac{3}{2}} (V^{(2)})^{-\frac{1}{2}} (N^{(2)})^{\frac{1}{2}} T^{\frac{3}{2}}$$

$$\therefore V^{(2)} = \frac{1}{27}\gamma^2 K^3 T^3 N\left(\varepsilon - k_B T \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)\right)^{-2}$$

$$V^{(1)} = V - V^{(2)}$$
$$= V - \frac{1}{27}\gamma^2 K^3 T^3 N\left(\varepsilon - k_B T \ln\left(\exp\left(\frac{\varepsilon}{k_B T}\right) - 1\right)\right)^{-2}$$

(14.1)

$$S = S_0 + RN \ln\left[\left(\frac{U + aN^2/V}{U_0}\right)^c \left(\frac{V - bN}{V_0}\right)\right]$$

$$\Delta S = \int \frac{d'Q}{T} = 0$$

$$\therefore S_1 = S_2$$

$$\therefore \left(\frac{U_1 + aN^2/V_1}{U_0}\right)^c \left(\frac{V_1 - bN}{V_0}\right) = \left(\frac{U_2 + aN^2/V_2}{U_0}\right)^c \left(\frac{V_2 - bN}{V_0}\right)$$

$$\therefore \frac{U_2 + aN^2/V_2}{U_1 + aN^2/V_1} = \left(\frac{V_1 - bN}{V_2 - bN}\right)^{\frac{1}{c}}$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{cRN}{U + aN^2/V}$$

$$\therefore \frac{T_2}{T_1} = \frac{U_2 + aN^2/V_2}{U_1 + aN^2/V_1} = \left(\frac{V_1 - bN}{V_2 - bN}\right)^{1/c} > 1$$

No. Date

(14.2)

$$S = S_0 + RN \ln\left[\left(\frac{U + aN^2/V}{V_0}\right)^c \left(\frac{V - bN}{V_0}\right)\right] \text{ より}$$

$$S_2 - S_1 = RN \ln\left[\left(\frac{V_2 - bN}{V_1 - bN}\right)\left(\frac{U + aN^2/V_2}{U + aN^2/V_1}\right)^c\right] > 0$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{cRN}{U + aN^2/V} \quad \therefore T = \frac{U + aN^2/V}{cRN}$$

$$\therefore T_2 - T_1 = \frac{aN^2}{cRN}\left(\frac{1}{V_2} - \frac{1}{V_1}\right) < 0$$